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Darcy-Forchheimer mixed convection heat and mass transfer in fluid saturated porous media

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Abstract *The Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium under the coupled effects of thermal and mass diffusion is analyzed on the basis of boundary-layer approximations. Similarity solutions are obtained for the case of constant surface temperature and concentration. Numerical results are presented for the distribution of velocity, temperature and concentration profiles within the boundary layer. Representative heat and mass transfer rates in terms of Nusselt and Sherwood numbers for various governing parameters are presented and discussed.*

Nomenclature

- C = concentration
- c_F = Forchheimer coefficient
- D = mass diffusivity
- f = dimensionless stream function
- f' = Blasius velocity
- g = gravitational acceleration
- K_1 = Darcy permeability
- K_2 = $c_F\sqrt{K_1}$
- k = thermal conductivity
- Le = Lewis number, α/D
- m'' = local mass flux
- N = buoyancy ratio
- Nu_x = local Nusselt number
- Pe_x = local Peclet number, $u_\infty x/\alpha$
- q'' = local heat flux
- Ra_x = thermal Rayleigh number, $(K_1 g \beta_T (T_w - T_\infty) x)/(\alpha\nu)$
- Sh_x = local Sherwood number
- T = temperature
- u, v = x and y velocity components
- x, y = Cartesian coordinates

Greek symbols

- α = thermal diffusivity of fluid-saturated porous media
- β_T = coefficient of thermal expansion
- β_C = coefficient of concentration expansion
- η = similarity variable
- ϕ = dimensionless concentration
- θ = dimensionless temperature
- ν = kinematic viscosity
- ψ = stream function
- Λ = inertia parameter, $K_2 u_\infty/\nu$

Subscripts

- w = wall property
- ∞ = porous media bulk property

Introduction

Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy and species transport (Ingham and Pop, 1998; Nield and Bejan, 1999; Pop and Ingham, 2001).

Most of the research efforts (e.g. Nield, 1968; Bejan and Khair, 1985; Yucel, 1990; Lai and Kulacki, 1991) concerned free convection using Darcy's law, which states the volume-averaged velocity is proportional to the pressure gradient. The Darcy model is shown to be valid under the conditions of low velocities and small porosity. In many practical situations, the porous medium is bounded by an impermeable wall, has higher flow rates, and reveals non-uniform porosity distribution in the near wall region, thereby making Darcy's law inapplicable. To model the real physical situations better, it is therefore, necessary to include the non-Darcian effects in the analysis of convective transport in a porous medium.

The mixed convection on bodies embedded in a non-Darcian porous medium have been extensively studied and reported for flow driven by temperature variations only (Lai and Kulacki, 1988, 1991; Ramaniah and Malarvizhi, 1990; Takhar *et al.*, 1990; Chen and Chen, 1990; Nakayama and Pop, 1991; Yu *et al.*, 1991; Shenoy, 1993; Nakayama, 1994; Kodah and Duwairi, 1996; Chen *et al.*, 1996; Chen, 1997; Takhar and Beg, 1997; Tashtoush and Kodah, 1998). Under the boundary layer and Darcy's approximations, the problem of coupled heat and mass transfer by mixed convection from a vertical flat plate in a saturated porous medium has been investigated by Lai (1991) and Yih (1998).

The objective of this paper is to consider simultaneous heat and mass transfer by Darcy-Forchheimer mixed convection from a flat plate embedded in a porous medium. A finite difference technique is used to solve the system of similar equations. The effects of the governing parameters on the heat and mass transfer are presented.

Analysis

We consider the mixed convection flow in a porous medium saturated with a Newtonian fluid bounded by a vertical flat plate with constant wall temperature T_w and constant wall concentration C_w aligned parallel to uniform free stream velocity u_∞ and temperature T_∞. The x-coordinate is measured along the plate from its leading edge, and y-coordinate normal to it. In the analysis the convecting fluid and the porous medium are everywhere in local thermodynamic equilibrium; the porous medium is isotropic and homogeneous; properties of the fluid and the porous medium are constants; the flow is laminar, steady state and two-dimensional and the boundary-layer and Boussinesq approximations hold. With these assumptions, the governing equations for non-Darcy mixed convective flow describing the conservation of mass, momentum, energy and concentration can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial y}+\frac{K_2}{\nu}\frac{\partial(u^2)}{\partial y}=\pm\frac{gK_1}{\nu}\left(\beta_T\frac{\partial T}{\partial y}+\beta_C\frac{\partial C}{\partial y}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}. \tag{4}$$

The boundary conditions are:

$$\left.\begin{array}{l}\text{at } y=0:\ \ v=0, T=T_w, C=C_w\\ \text{as } y\to\infty:\ \ u=u_\infty, T=T_\infty, C=C_\infty\end{array}\right\}. \tag{5}$$

The ± sign in equation (2) is used to indicate the direction of natural convection flow. The + sign corresponds to the case where the buoyancy force has a component "aiding" the forced flow, and the – sign to the "opposing" case.

Defining the stream function ψ such that:

$$u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}, \tag{6}$$

and thereby satisfying the continuity equation (1), equations (2)-(4) transform to:

$$\frac{\partial^2\psi}{\partial y^2}+\frac{K_2}{\nu}\frac{\partial}{\partial y}\left[\left(\frac{\partial\psi}{\partial y}\right)^2\right]=\pm\frac{gK_1}{\nu}\left[\beta_T\frac{\partial T}{\partial y}+\beta_c\frac{\partial C}{\partial y}\right], \tag{7}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}. \tag{9}$$

The partial differential equations (7)-(9) are transformed by means of the appropriate similarity variables for mixed convection:

$$\eta=Pe_x^{\frac{1}{2}}\frac{y}{x}, \tag{10}$$

$$\psi=\alpha Pe_x^{\frac{1}{2}}f(\eta), \tag{11}$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{12}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \tag{13}$$

The resulting equations are:

$$f'' + 2\Lambda f' \, f'' = \pm \frac{Ra_x}{Pe_x}(\theta' + N\phi'), \tag{14}$$

$$\theta'' + \frac{1}{2} f \, \theta' = 0, \tag{15}$$

$$\phi'' + \frac{1}{2} Le \; f \; \phi' = 0, \tag{16}$$

where $\Lambda = (K2u_\infty)/\nu$ is the inertia parameter. Ra_x/Pe_x is a parameter for comparing the intensities of natural and forced convection effects. In fact, when $Ra_x/Pe_x \gg 1$ flow is dominated by natural convection, where when $Ra_x/Pe_x \ll 1$ forced convection takes the leading role. Thus, when $Ra_x/Pe_x = 1$, the effects of natural and forced convection achieve equal importance and flow is truly under mixed convection conditions.

The corresponding transformed boundary conditions are:

$$\text{at } \eta = 0 : f =, \theta = 1, \phi = 1, \tag{17}$$

$$\text{at } \eta \to \infty : f' \to 1, \theta \to 0, \phi \to 0. \tag{18}$$

In the above equations, the prime denotes differentiation with respect to η. Of special importance for this flow situation are the local Nusselt number and the local Sherwood number. These are defined as follows:

$$Nu_x = \frac{q''_{y=0} \; x}{(T_w - T_\infty) \; k} = -\,\theta'(0)Pe_x^{1/2}, \tag{19}$$

$$Sh_x = \frac{m''_{y=0} \; x}{(C_w - C_\infty) \; D} = -\,\phi'(0)Pe_x^{1/2}. \tag{20}$$

Results and discussion

The system of equations (14)-(16), together with their corresponding boundary conditions, equations (17) and (18), is solved numerically using a variable step size finite-difference method. The basic discretization is the trapezoidal rule over a non-uniform mesh. Global error estimates are produced to control the computation. The resulting non-linear algebraic system is solved by Newton's method with step control.

Figures 1 and 2 present the trend of the temperature, concentration and velocity profiles within the boundary layer as functions of Λ and Ra_x/Pe_x, respectively. The porous media inertial resistance described by Λ tends to decrease the velocity and create more uniform velocity field. This is because the form drag of the porous medium is increased when the inertia effect is included. Figure 1 shows that as Λ increases the temperature and concentration increase, and hence the inertia effect tends to thicken the thermal and concentration boundary layers. As Figure 2 indicates, with increasing Ra_x/Pe_x we observe that the velocity increases while the temperature and concentration decrease.

Figures 3-6 show the local Nusselt and Sherwood numbers as functions of the inertia parameter Λ for various values of N (positive and negative values) and Le. It is apparent that lower heat and mass transfer rates occur as Λ increases. This is clear from the fact that inertia effects tend to slow down the buoyancy-induced flow in the boundary layer (Figure 1(c)) and so retard the heat and mass transfer rates. It is also observed that the inertia effect is more pronounced at higher values of $|N|$ and lower values of Le. Moreover, for fixed values of Ra_x/Pe_x and Le, all curves corresponding to different values of $|N|$ are seen to converge to one point at higher values of Λ.

The heat and mass transfer results in terms of $Nu_x/Pe_x^{1/2}$ and $Sh_x/Pex_x^{1/2}$ as functions of the mixed convection parameter Ra_x/Pe_x for both aiding and opposing flows are displayed in Figures 7-12. For aiding flow (Figures 7-10), It is seen that for particular values of Λ, N and Le, the heat and mass transfer rates increase with an increase in Ra_x/Pe_x. This because increasing Ra_x/Pe_x increases the momentum transport in the boundary layer (Figure 2(c)) and more heat and species are carried out of the surface, thus decreasing the thickness of the thermal and concentration boundary layers (Figures 2(a) and 2(b)) and hence increasing the heat and mass transfer rates. Moreover, when the mixed convection parameter Ra_x/Pe_x is very small the values of $Nu_x/Pe_x^{1/2}$ and $Sh_x/Pex_x^{1/2}$ corresponding to different values of N approach individually to one point in the forced convection regime.

Figures 7-10 also indicate that for aiding flow (i.e. + ve Ra_x/Pe_x) the effect of buoyancy ratio $|N|$ is to increase the surface heat and mass transfer rates. This can be attributed to the fact that increasing $|N|$ increases the vertical velocity and decreases the thickness of the temperature and concentration boundary layers. Therefore, the temperature and concentration gradients are increased and, hence, so are heat and mass transfer rates. Approaching the forced convection dominated regime, i.e. $Ra_x/Pe_x = 0.1$, the heat and mass transfer results show little dependence on N. Furthermore, the buoyancy ratio has a more pronounced effect on the Nusselt and Sherwood numbers at lower values of Le. As observed from Figures 11 and 12, for opposing flows (i.e. – ve Ra_x/Pe_x), heat and mass transfer rates are decreased as N is increased.

It is worth pointing out that the mass transfer coefficient does not have any physical meaning for thermally driven flows, i.e. $N = 0$. The curves for $N = 0$ are not included in Figures 5, 9 and 12.

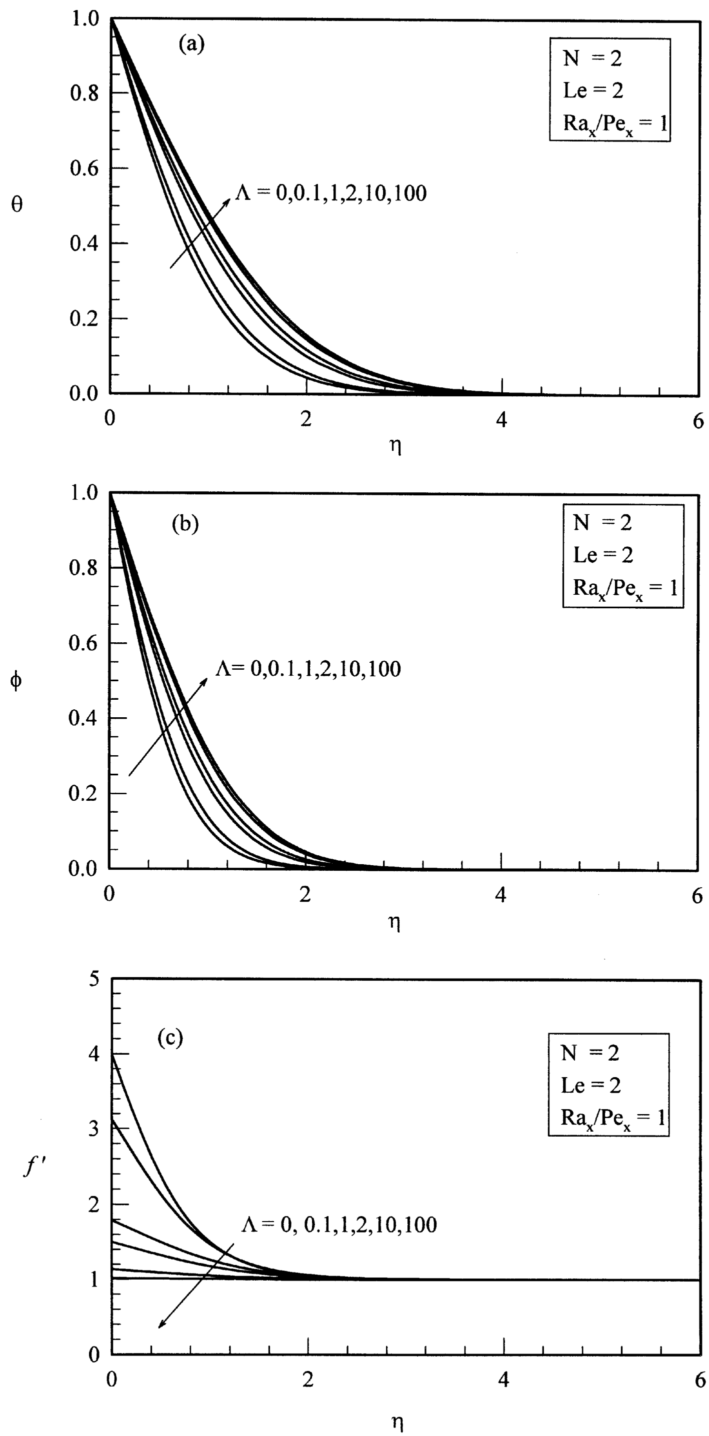

Figure 1.
Effect of intertia parameter (Λ) on temperature, concentration, and velocity profiles

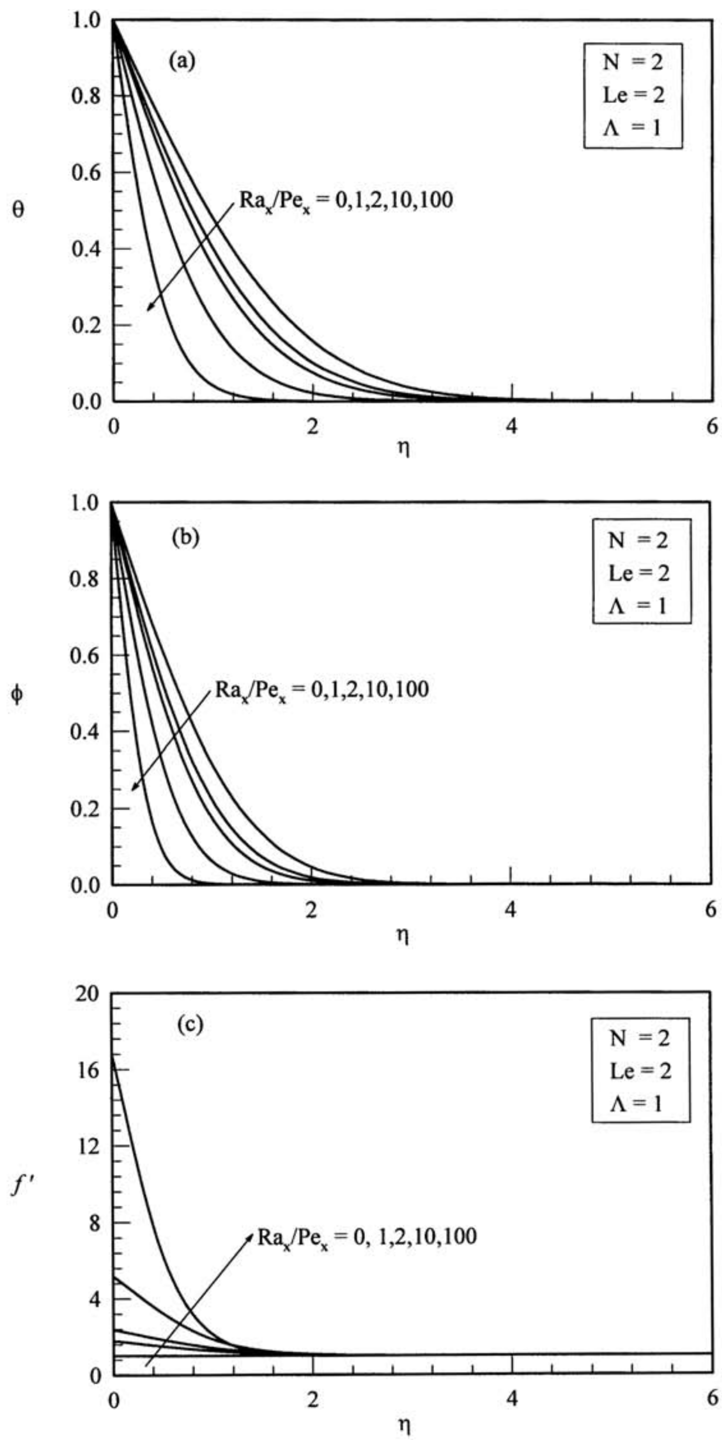

Figure 2.
Effect of Ra_x/Pe_x on temperature, concentration, and velocity profiles

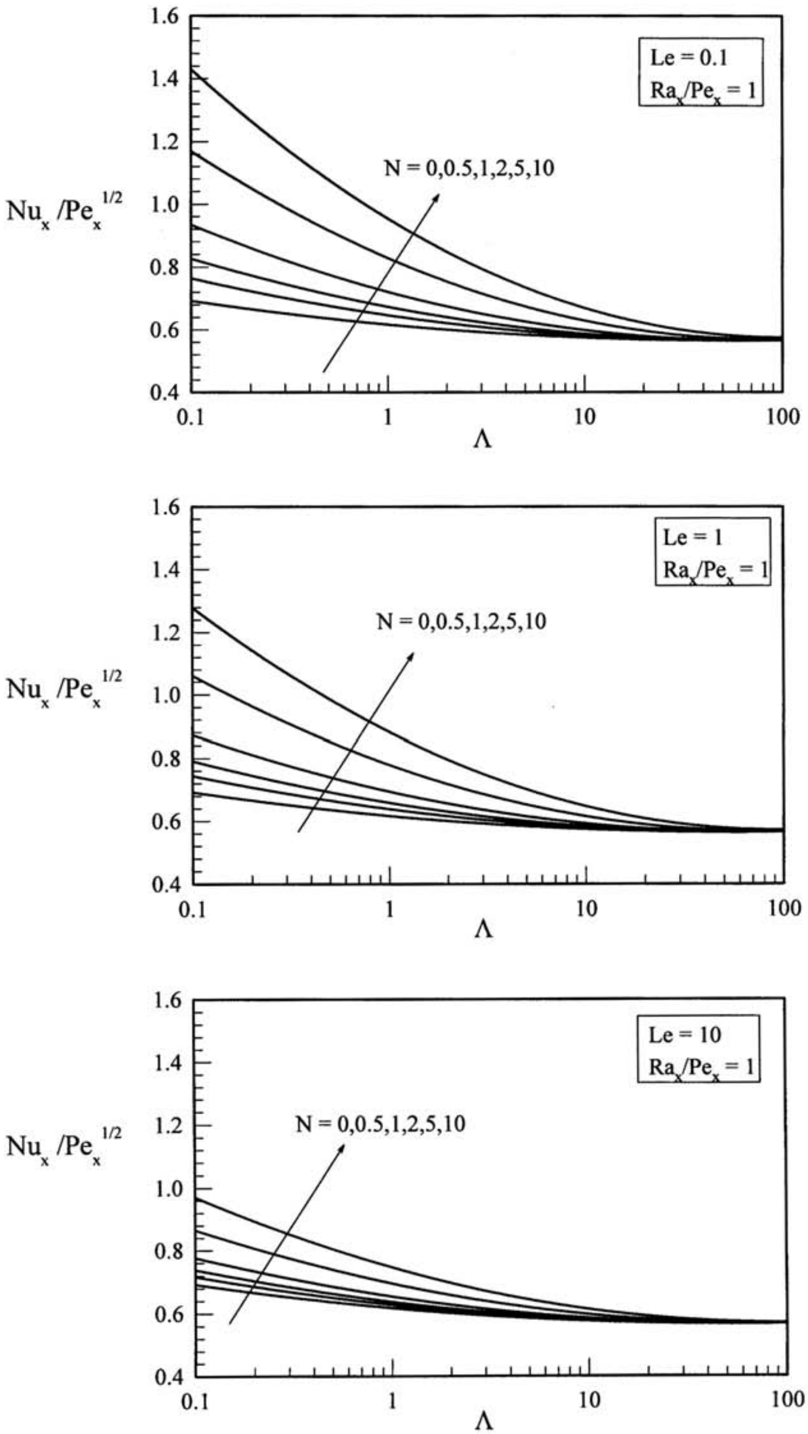

Figure 3.
Local Nusselt number as a function of the inertia parameter (Λ) for positive N values

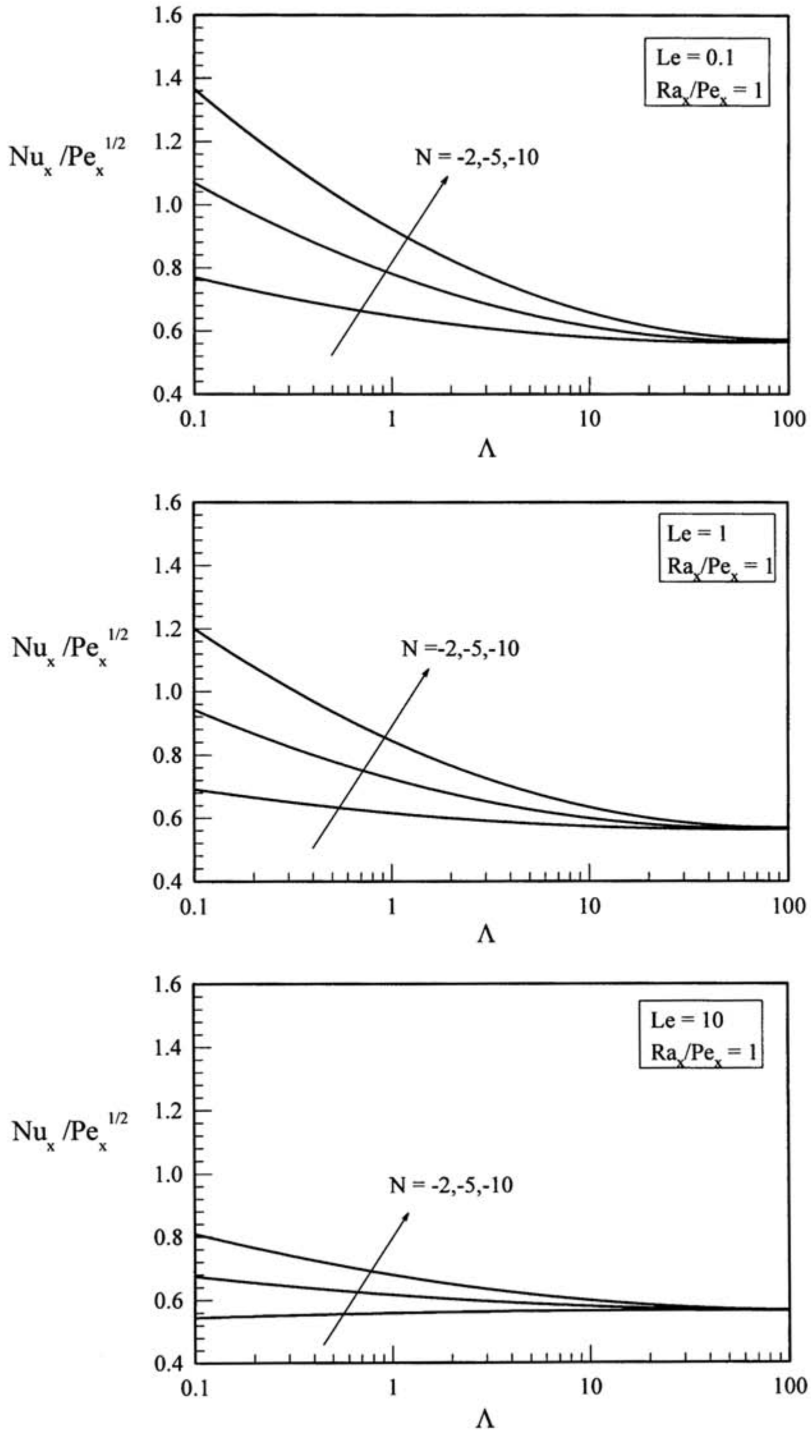

Figure 4.
Local Nusselt number as a function of the inertia parameter (Λ) for negative N values

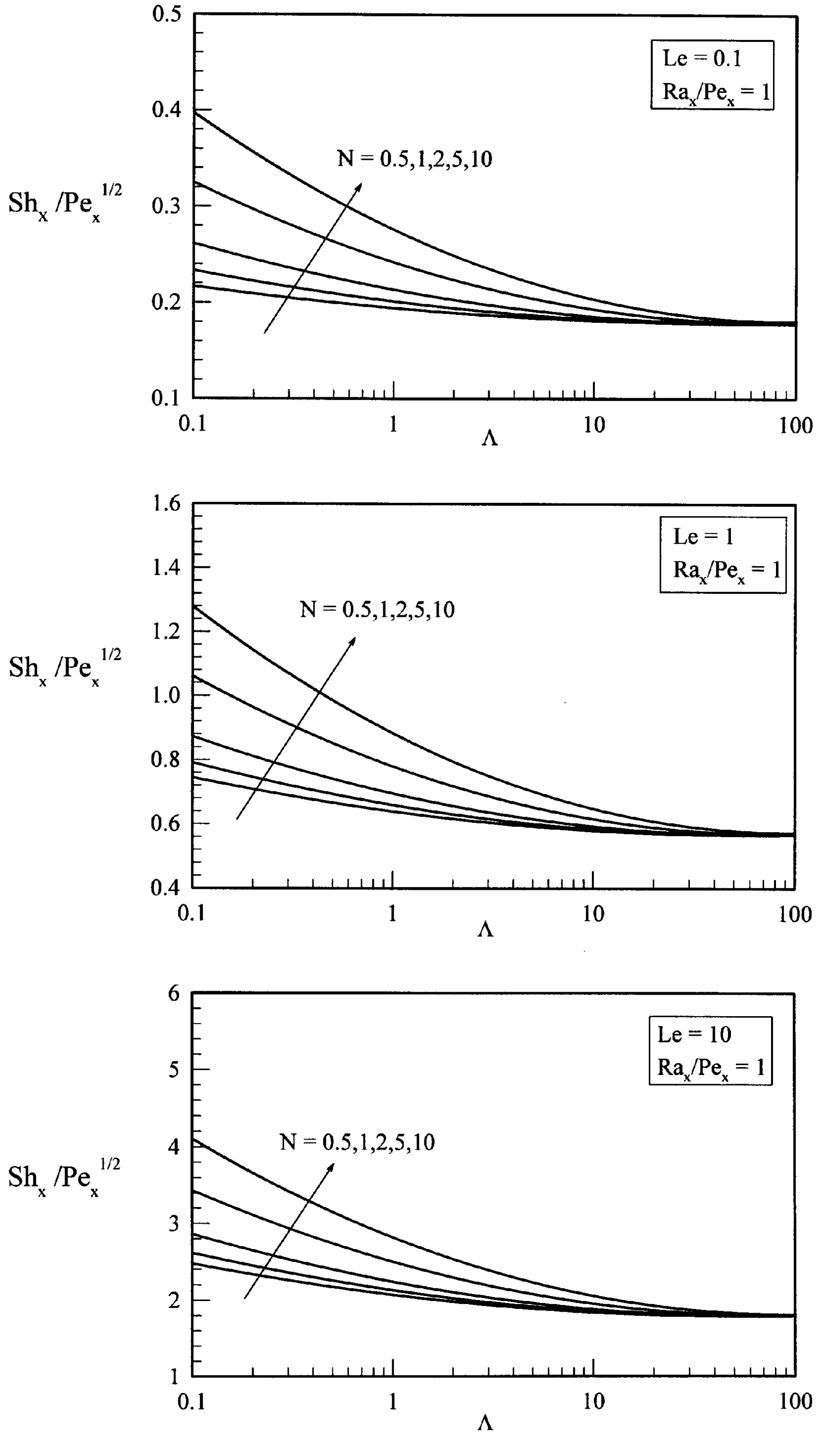

Figure 5.
Local Sherwood number as a function of the inertia parameter (Λ) for positive N values

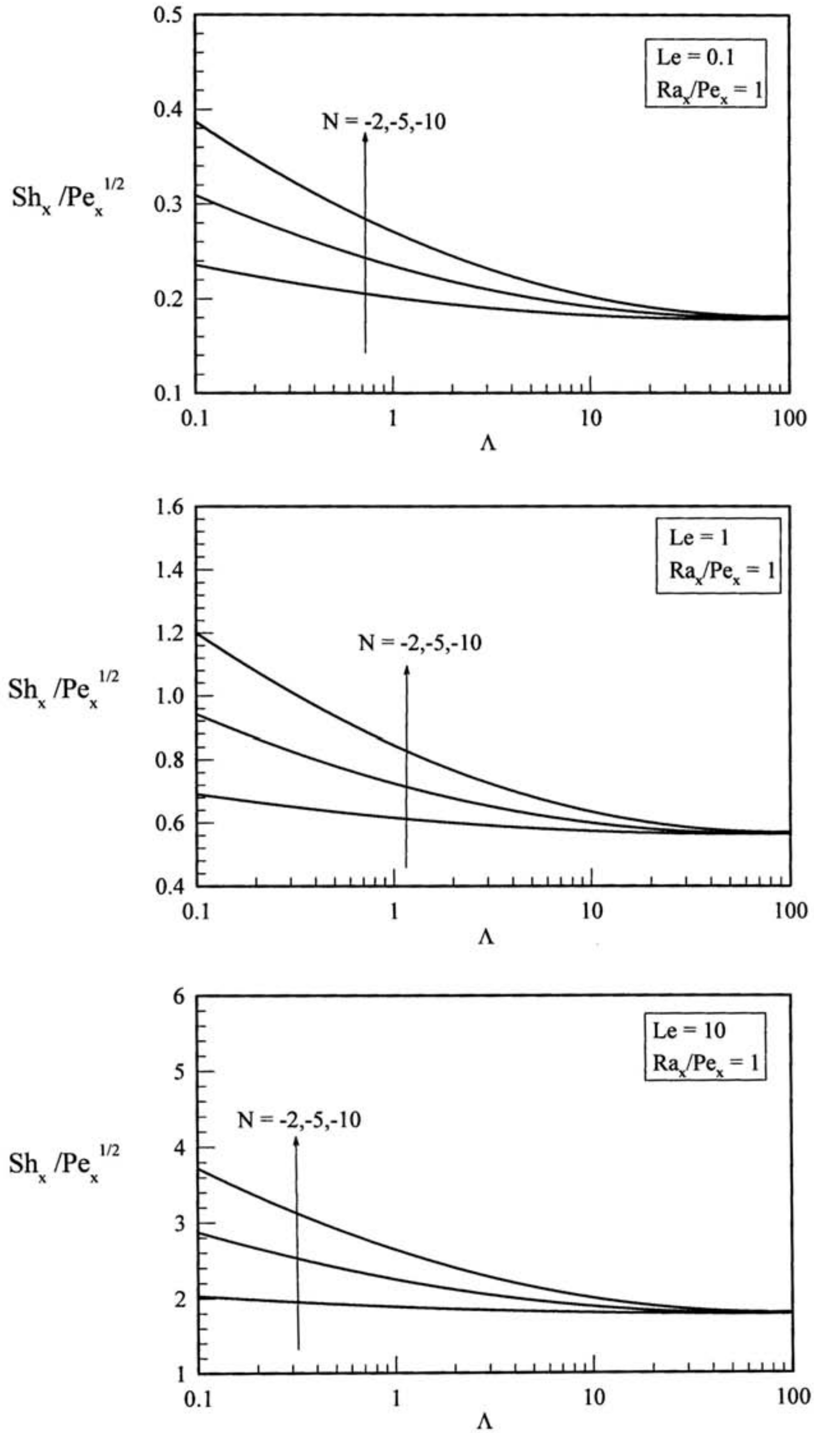

Figure 6.
Local Sherwood number as a function of the inertia parameter (Λ) for negative N values

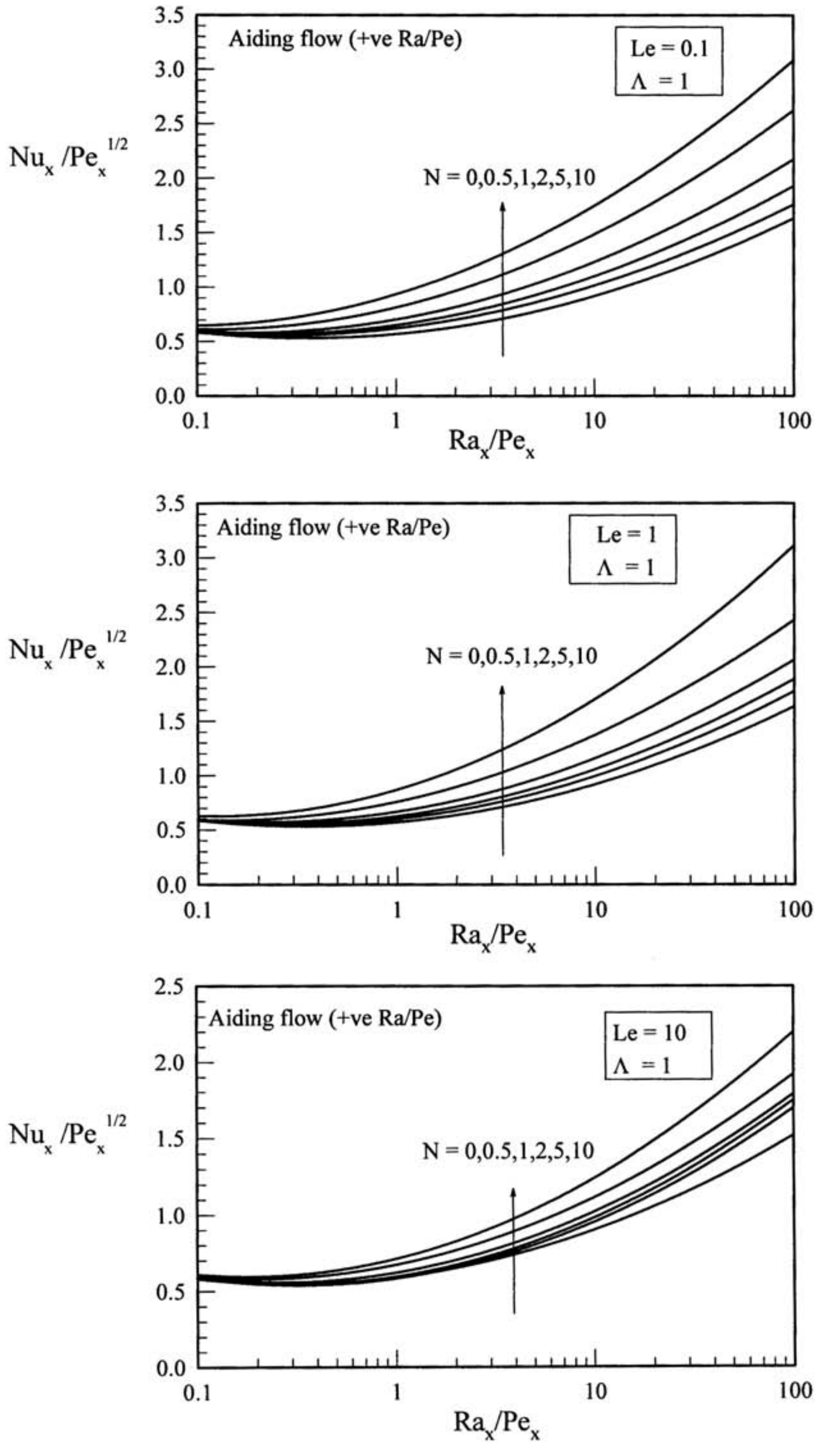

Figure 7.
Local Nusselt number as a function of the mixed convection parameter (Ra_x/Pe_x) for aiding flow and positive N values

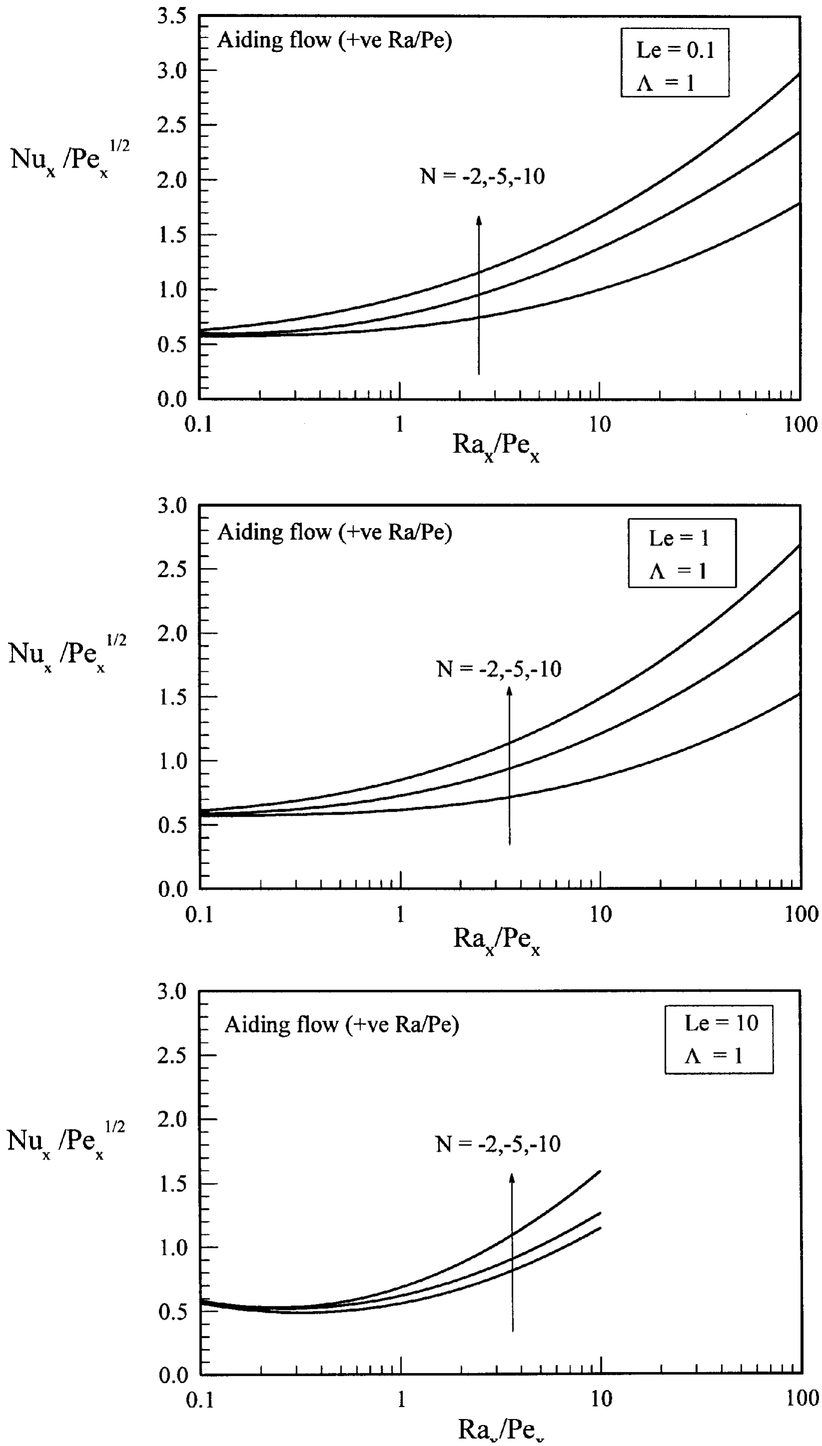

Figure 8.
Local Nusselt number as a function of the mixed convection parameter (Ra_x/Pe_x) for aiding flow and negative N values

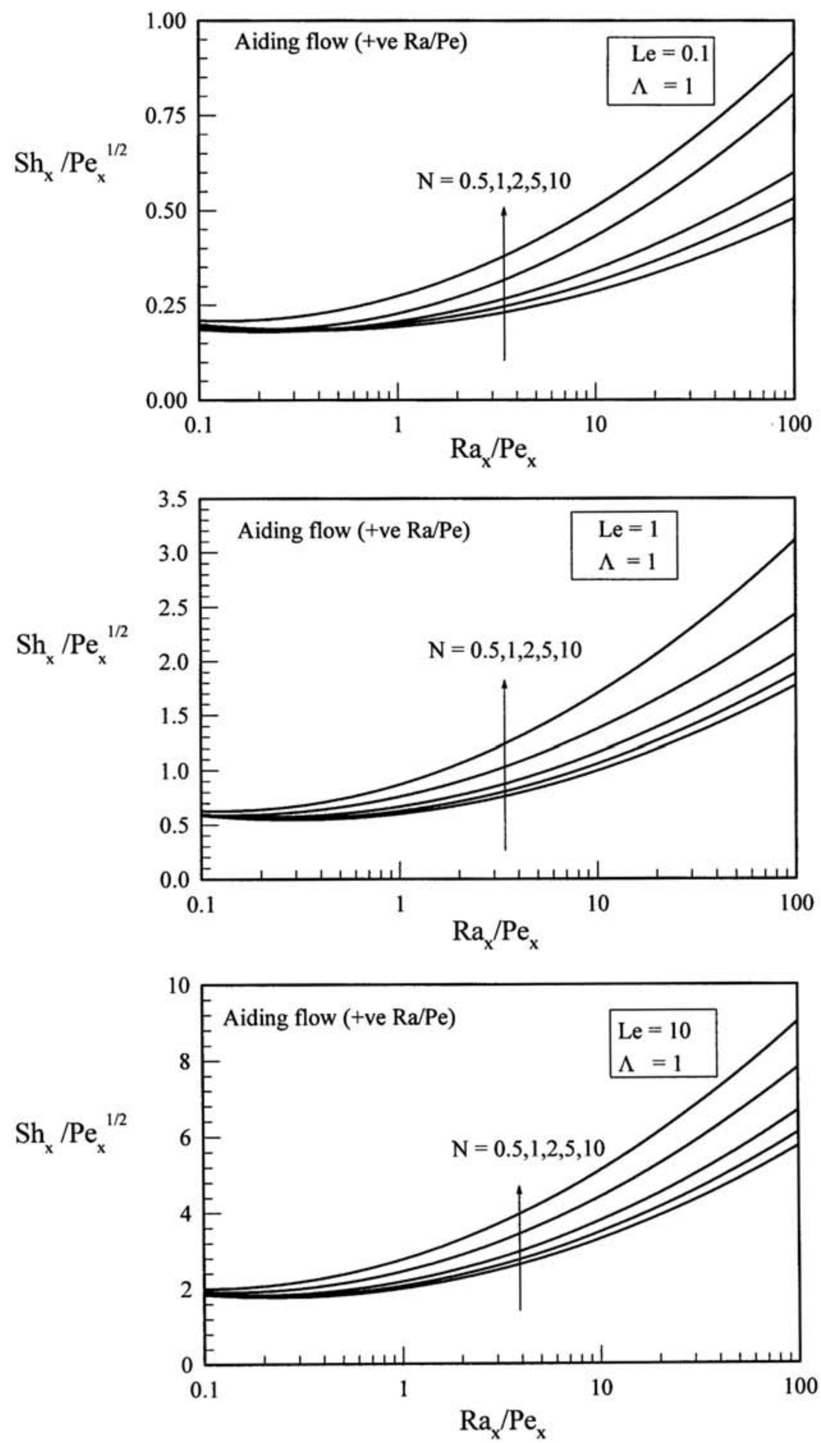

Figure 9.
Local Sherwood number as a function of the mixed convection parameter (Ra_x/Pe_x) for aiding flow and positive N values

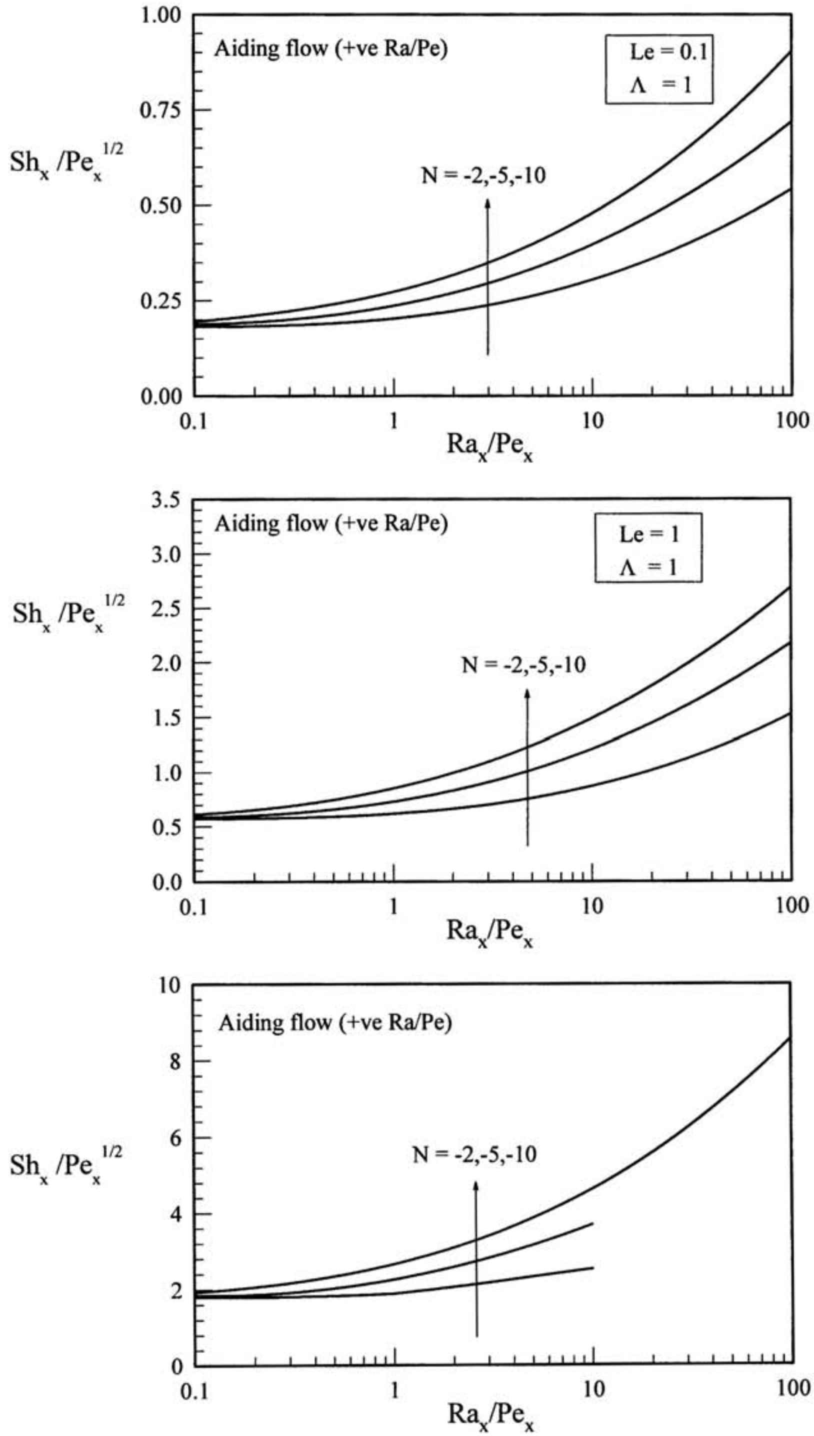

Figure 10.
Local Sherwood number as a function of the mixed convection parameter (Ra_x/Pe_x) for aiding flow and negative N values

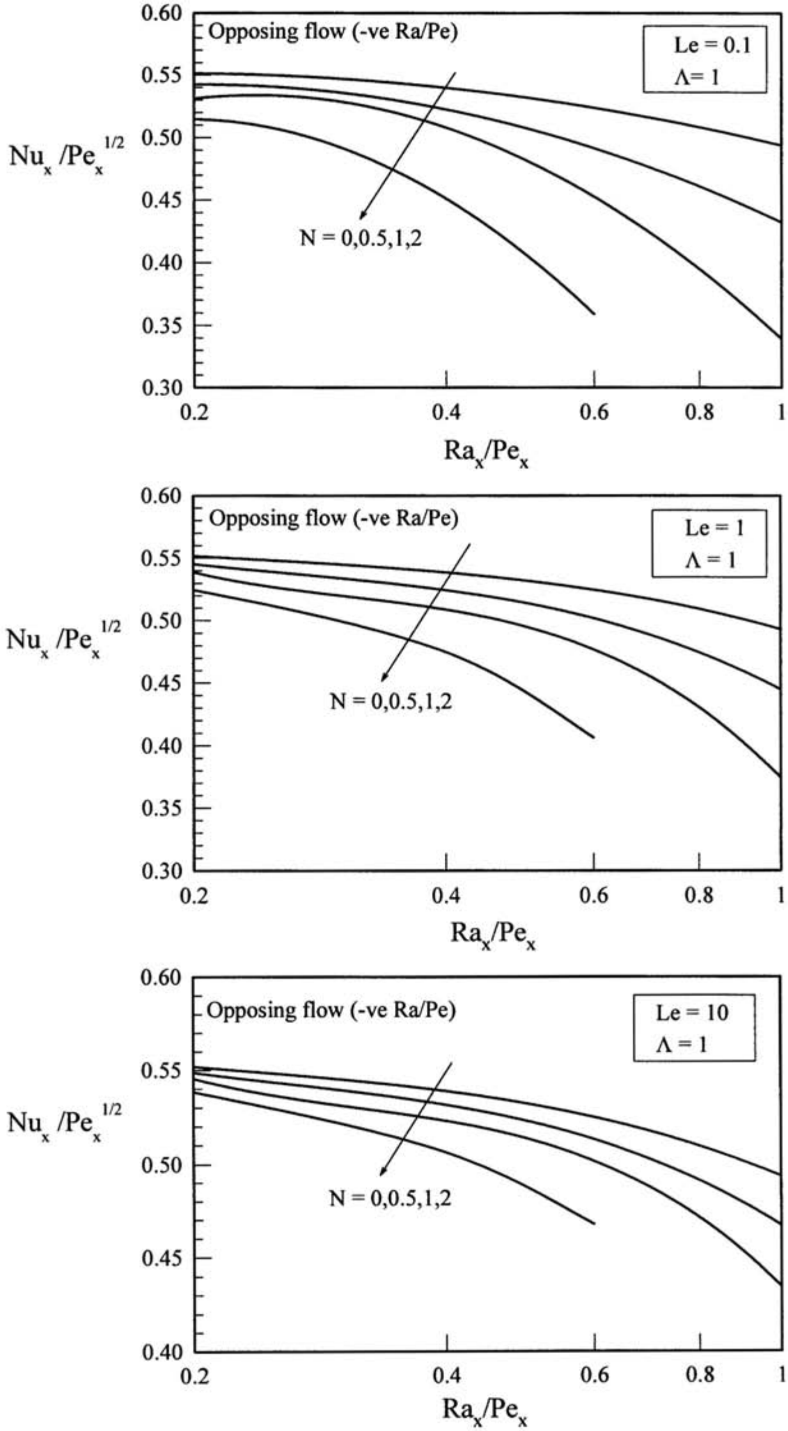

Figure 11.
Local Nusselt number as a function of mixed convection parameter (Ra_x/Pe_x) for opposing flow

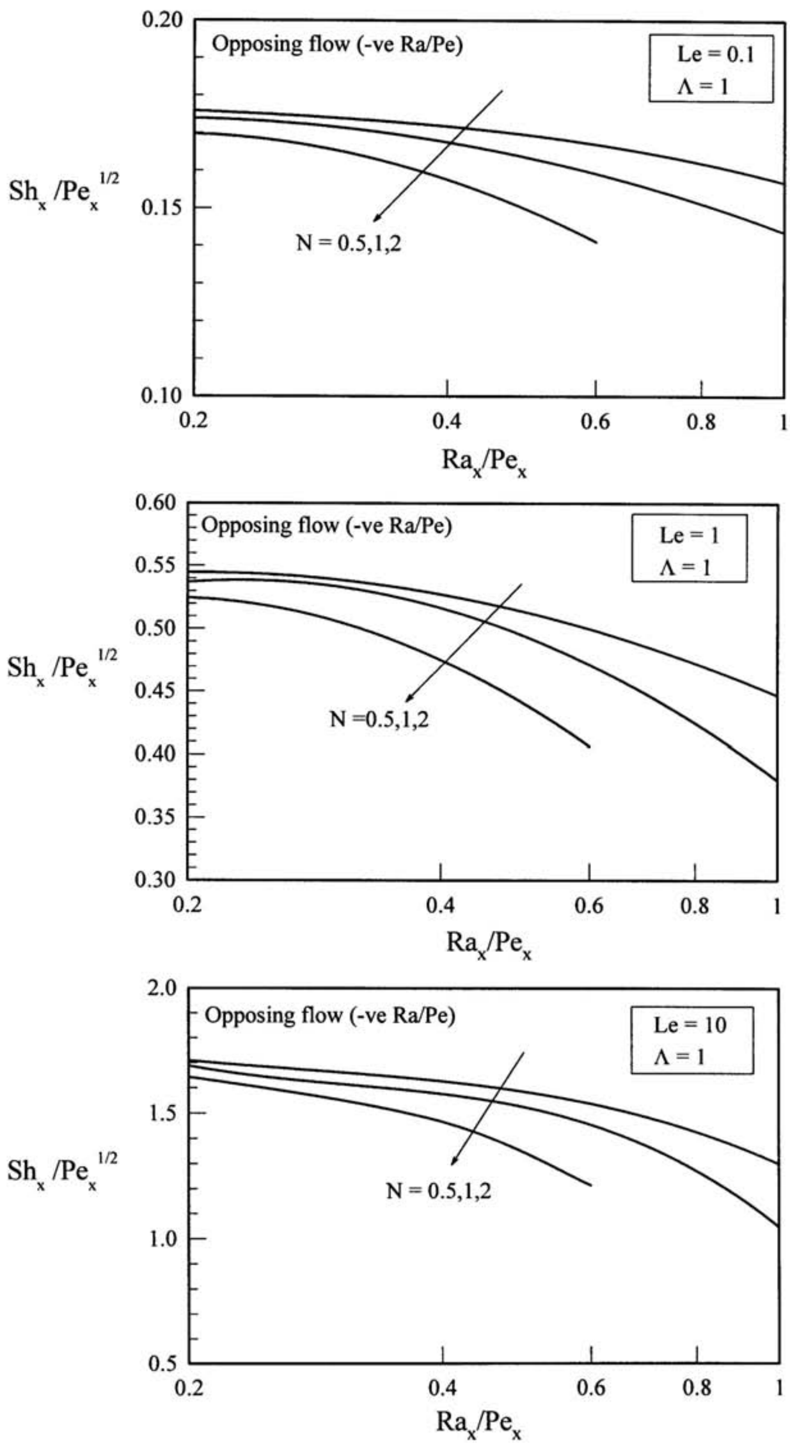

Figure 12.
Local Sherwood number as a function of mixed convection parameter (Ra_x/Pe_x) for opposing flow

It is apparent also that increasing the Lewis number increases the Sherwood number and decreases the Nusselt number. This is because a greater value of Le corresponds to smaller mass diffusivities and hence a thinner concentration boundary layer relative to the thermal boundary layer, thereby resulting in a greater concentration gradient and a smaller temperature gradient, which in tern enhances the mass transfer rate and reduces the heat transfer rate.

Conclusion

In this paper, we have presented an analysis for the problem of Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium under the coupled effects of temperature and concentration variations. Numerical results are presented for the velocity, temperature and concentration as well as the Nusselt number and Sherwood number variations with the mixed convection parameter, the inertia parameter, the buoyancy ratio, and the Lewis number.

References

Bejan, A. and Khair, K.R. (1985), "Heat and mass transfer by natural convection in a porous medium", *Int. J. Heat Mass Transfer*, Vol. 29, pp. 909-18.

Chen, C.H. (1997), "Non-Darcian mixed convection over a vertical flat plate in porous media with variable wall heat flux", *Int. Comm. Heat Mass Transfer*, Vol. 24, pp. 427-37.

Chen, C.H. and Chen, C.K. (1990), "Non-Darcian mixed convection along a vertical plate embedded in a porous medium", *Appl. Math. Modelling*, Vol. 14, pp. 482-8.

Chen, C.H., Chen, T.S. and Chen, C.K. (1996), "Non-Darcian mixed convection along nonisothermal vertical surfaces in porous media", *Int. J. Heat Mass Transfer*, Vol. 39, pp. 1157-64.

Ingham, D.B. and Pop, I. (1998) (Eds), *Transport Phenomena in Porous Media*, Pergamon, Oxford.

Kodah, Z.H. and Duwairi, H.M. (1996), "Inertia effects on mixed convection for vertical plates with variable wall temperature in saturated porous media", *Heat Mass Transfer*, Vol. 31, pp. 333-8.

Lai, F.C. (1991), "Coupled heat and mass transfer by mixed convection from a vertical plate in a saturated porous medium", *Int. Comm. Heat Mass Transfer*, Vol. 18, pp. 93-106.

Lai, F.C. and Kulacki, F.A. (1988), "Effects of flow inertia on mixed convection along a vertical surface in a saturated porous medium", *ASME HTD 96*, Vol. 1, pp. 643-52.

Lai, F.C. and Kulacki, F.A. (1991), "Non-Darcy mixed convection along a vertical wall in a saturated porous medium", *ASME J. Heat Transfer*, Vol. 34, pp. 887-90.

Nakayama, A. (1994), "A unified theory for non-Darcy free, forced and mixed convection problems associated with a horizontal line heat source in a porous medium", *ASME J. Heat Transfer*, Vol. 116, pp. 508-13.

Nakayama, A. and Pop, I. (1991), "A unified similarity transformation for free, forced and mixed convection in Darcy and non-Darcy porous media", *Int. J. Heat Mass Transfer*, Vol. 34, pp. 357-67.

Nield, D.A. (1968), "Onset of thermohaline convection in porous medium", *Water Resources Res.*, Vol. 4, pp. 553-60.

Nield, D.A. and Bejan A. (1999), *Convection in Porous Media*, 2nd ed., Springer, New York, NY.

Pop, I. and Ingham, D.B. (2001), *Convection Heat Transfer: Mathematical and Computational Modelling of Viscous Fluids and Porous Media*, Pergamon, Oxford.

Ramaniah, G. and Malarvizhi, G. (1990), "Non-Darcy regime mixed convection on vertical plates in saturated porous media with lateral mass flux", *Acta Mech.*, Vol. 81, pp. 191-200.

Shenoy, A.V. (1993), "Darcy-Forchheimer natural, forced and mixed convection heat transfer in non-Newtonian power-law fluid saturated porous media", *Transport in Porous Media*, Vol. 11, pp. 219-41.

Takhar, H.S. and Beg, O.A. (1997), "Non-Darcy effects on convective Boundar-layer flow past a semi-infinite vertical plate in saturated porous media", *Heat Mass Transfer*, Vol. 32, pp. 33-4.

Takhar, H.S., Soundalgekar, V.M. and Gupta, A.S. (1990), "Mixed convection of an incompressible viscous fluid in a porous medium past a hot vertical plate", *Int. J. Non-Linear Mech.*, Vol. 25, pp. 723-8.

Tashtoush, B. and Kodah, Z. (1998), "No slip boundary effects in non-Darcian mixed convection from a vertical wall in saturated porous media: analytical solution", *Heat Mass Transfer*, Vol. 34, pp. 35-9.

Yih, K.A. (1998), "Coupled heat and mass transfer in mixed convection over a vertical flat plate embedded in saturated porous media: PST/PSC or PHF/PMF", *Heat Mass Transfer*, Vol. 34, pp. 55-61.

Yu, W.S., Lin, H.T. and Lu, C.S. (1991), "Universal formulations and comprehensive correlations for non-Darcy natural convection and mixed convection in porous media", *Int. J. Heat Mass Transfer*, Vol. 34, pp. 2859-68.

Yucel, A. (1990), "Natural convection heat and mass transfer along a vertical cylinder in a porous medium", *Int. J. Heat Mass Transfer*, Vol. 33, pp. 2265-74.